MAD 4401 FINAL EXAM

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NAME: ________________________________

Work all problems and show all work. Each problem is worth ten points. Partial credit will be given for correct reasoning. Partial credit will be deducted for incorrect statements and reasoning.

Problem 1. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 2. Solve the equation $x \cdot \exp(x) = 3$ by the Newton-Raphson method. Give your starting point and the iterates of the Newton function. Give your final solution to ten digits.

Problem 3. Determine the polynomial $p(x)$ of degree 4 passing through the points $\left\{(0,0)\,,(1,0)\,,\left(\frac{3}{2},1\right),(2,0)\,,\left(\frac{7}{2},0\right)\right\}$.

Problem 4. Determine the normalized closed Newton-Cotes coefficients for five points, $\{a_0, a_1, \ldots, a_4\}$.

Problem 5. Use the Romberg method for approximating the integral to estimate the integral $\int_1^2 \sin(x^3)dx$. Use thirty-two points (i.e., $n = 5$). Indicate the estimate of the integral by the method and how many digits are likely correct. Explain.

Problem 6. Determine the coefficients to compute the second derivative of $f(x) = \sin(x^3)$ at $a = 2$ using the points $\{a-h, a, a+h, a+2h\}$. Give the estimate of the second derivative as a function of h. Determine the best value of h for the greatest accuracy of the answer. How many digits accuracy can you expect with this choice of h?

Problem 7. Give a numerical solution of the differential equation $\frac{dx}{dt} = f(t, x) = t^2 \cdot x$ with $x(0) = -1$. Solve using the Taylor method of order three. Solve on the interval $[0, 1]$ using $h = \frac{1}{4}$ and $n = 4$.

Problem 8. Determine the steady-state probabilities for an M/M/1/FIFO queue. Assume that the arrival rate is α, the service rate is σ, and that $\sigma > \alpha > 0$.

Problem 9. Consider a medical test for a certain disease. Suppose that the test is positive with probability .97 if the person has the disease. Suppose that the test is negative with probability .93 if the person does not have the disease. Suppose that the disease occurs with probability .05 in the general population. Suppose that a person is selected at random and tested and the test is positive. What is the probability that the person has the disease?

Problem 10. Do four iterations of the **Picard Method** for the differential equation $\frac{dx}{dt} = \sin(t) \cdot x$ with $x(0) = 1$.

$x_0 =$

$x_1 =$

$x_2 =$

$x_3 =$

$x_4 =$