

FALL 2019 FINAL EXAM

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NAME _____

Work all problems and show all work. Each problem is worth ten points. Partial credit will be given for correct work even though the answer may be wrong. Points will be taken off for incorrect reasoning even though the final answer may be right.

Problem 1. Determine a function $p(h)$ to estimate the derivative of $\sin(x^2)$ at $x = 1$.

$$\left. \frac{df}{dx} \right|_{x=a} \approx p(h) = A_0 \cdot f(a - 3h) + A_1 \cdot f(a - h) + A_2 \cdot f(a) + A_3 \cdot f(a + h) + A_4 \cdot f(a + 5h)$$

Problem 2. For a queue of type M/M/ ∞ what is the average number in the queue with arrival rate α and service rate σ ? What is the long-term probability, p_n , of n in the system?

Problem 3. Consider a queue of the form M/M/2/FIFO. Suppose that $\alpha = 10$ and $\sigma = 10$. Estimate the average number in the system for this queue. Explain.

Problem 4. Assume that human blood contains 30×10^{12} erythrocytes. Suppose that each erythrocyte lives 100 days. How many erythrocytes are being produced each day by the bone marrow?

Problem 5. Approximate the following differential equation using **linearode**.

$$\frac{d^2x}{dt^2} = -x \quad x(0) = 1 \quad \frac{dx}{dt}(0) = 0$$

Solve on the interval $[0, \pi]$ using five steps. Determine the Taylor expansion up to ten terms.

Problem 6. Solve the equation $x^5 + 4 = \cos x$ by the Newton-Raphson method. Give the Newton function. Find a starting point for which the method converges. Give the starting point and the iterations with twelve digits accuracy. Circle the final answer.

Problem 7. Determine the normalized coefficients for Newton-Cotes Integration with 5 intervals.

Problem 8. Estimate $\int_0^\pi \cos(x^2)dx$ using Romberg Integration using 2^7 subintervals. Give the first column of the result to 5 digits and the last two columns to 12 digits. Circle the best answer. How many digits would you say are correct?

Problem 9. Determine the points and coefficients to be used for Gaussian Quadrature with six points. What points and what coefficients should you use to estimate the following integral? $\int_{-2}^5 f(x)dx$

Problem 10. State the **Mean Value Theorem**. State the **Intermediate Value Theorem**. Give the **Taylor Expansion** for a function $f(x)$ centered at a .