PRACTICE PROBLEMS FOR TEST 2 - JAMES KEESLING

The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on Test 2.

Problem 1. Estimate $\frac{d^n f}{dx^n}$ at $x = a$ using the points $\{a - 5 \cdot h, a - 2 \cdot h, a - h, a, a + h, a + 2 \cdot h, a + 4 \cdot h\}$. Assume that $f(x) = \exp(x^2)$ and $a = 1$. Assume that the accuracy of the calculation of $f(x)$ is 10^{-24}. For which n can this be done? What is the best h? Estimate the error.

Problem 2. Solve the differential equation $\frac{dx}{dt} = f(t, x) = t^2 \cdot x$ with $x(0) = 2$. Solve using Picard iteration for five iterations. Solve using the Taylor method of order 5 using $h = \frac{1}{10}$ and $n = 10$.

Problem 3. Find a Taylor expansion for the solution $x(t) = a_0 + a_1 t + a_2 t^2 + \cdots$ for the differential equation $\frac{dx}{dt} = t \cdot x$ with the boundary condition $x(0) = 1$. Solve for $\{a_0, a_1, a_2, a_3, a_4, a_5\}$. Do this by hand solving for these coefficients recursively. Solve for the coefficients using the Taylor Method program included in your program collection. Can you determine the general a_n?

Problem 4. Simulate rolling ten dice using the **dice** program. Do this ten times, compute the sum of the dice each time, and record the results.

Problem 5. Use the **Queue** program to simulate a queueing system for $M/M/1/FIFO$ with $\alpha = 8$ per hour and $\sigma = 10$ per hour for a time period of 100 hours. Simulate a queueing system for $M/M/2/FIFO$ with $\alpha = 8$ per hour and $\sigma = 10$ per hour for a time period of 100 hours. How do the results compare with the theoretical calculations for $\{\overline{p}_n\}_{n=0}^{\infty}$ in each of these cases?