MAD 4401 PRACTICE TEST 1 FALL 2018 - JAMES KEESLING

NAME ______________________________

The typical test with have five problems with each problem worth 20 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Solve the equation $x^5 + 2 = \cos x$ by the Newton-Raphson method. Give the Newton function. Find a starting point for which the method converges. Give the starting point and the iterations with five digits accuracy. Give the final answer to twelve digits and circle the final answer. Solve the same problem using the Bisection Method.

Problem 2. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 3. Give the Legendre polynomial of degree 6. Determine the points and weights of for Gaussian quadrature for 6 points in the interval $[-1, 1]$.

Problem 4. Estimate $\int_0^\pi \cos(x^2)dx$ using Romberg Integration using 2^7 subintervals. Give the first column of the result to 5 digits and the last two columns to 12 digits. Circle the best answer.

Problem 5. Determine the Newton-Cotes coefficients using 4 subintervals. What points and what coefficients should you use to estimate the following integral? $\int_{-2}^5 f(x)dx$

Problem 6 Estimate the integral $\int_{-4}^4 \frac{1}{1+x^2}dx$ using Newton-Cotes with 14 intervals.

Problem 7 Estimate the integral $\int_{-4}^4 \frac{1}{1+x^2}dx$ using Gaussian Quadrature using 6 points.

Problem 8 State the **Mean Value Theorem**. State the **Intermediate Value Theorem**. Give the **Taylor Expansion** for a function $f(x)$ centered at a.

Problem 9 Estimate the first, second, and third derivative of $\sin(x^2)$ at $x = 1$ using the points $\{1 - 2h, 1 - h, 1, 1 + h, 1 + 3h\}$. Show what you think are the correct digits in each estimate. What is the best h to use?

Problem 10 Determine the Lagrange polynomial through the points $\{(0, 1), (1, -1), (2, -3), (3, 4), (4, -1)\}$.