MAD 4401 PRACTICE TEST 2 - JAMES KEESLING

The problems below are typical of what will be on Test 2. However, there will be only five problems on the test.

Problem 1. Determine the coefficients to compute the first, second and third derivative of $f(x) = \sin(x^2)$ at $a = 2$ using the points $\{a - 2h, a - h, a, a + h, a + 2h\}$. Give the estimate of the derivative as a function of h. Determine the best value of h for the greatest accuracy of the answer. How many digits accuracy can you expect with this choice of h?

Problem 2. Solve the differential equation for $\frac{dx}{dt} = f(t, x) = \cos(t) \cdot x$ with $x(0) = 1$. Solve using Picard iteration for five iterations. Solve using the Taylor method of order 3,4, and 5.

Problem 3. Solve the differential equation $\frac{d^2x}{dt^2} = -x$ with $x(0) = 1$ and $x'(0) = 1$. Solve on the interval $[0, 1]$ with $h = \frac{1}{10}$ and $n = 10$.

Problem 4. Solve $\frac{dx}{dt} = M \cdot x$ with $M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and with $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Problem 5. Assume a queueing system with Poisson arrival rate of α and a single server with an exponential service rate σ. Assume that $\sigma > \alpha > 0$. This is an M/M/1/FIFO queue. Determine the steady-state probabilities for n, $\{\overline{p}_n\}_{n=0}^{\infty}$ for this system.

Problem 6. Suppose that there are an infinite number of servers in the queueing system M/M/∞. Suppose that the arrival rate is α and the service rate for each server is σ. Determine the steady-state probabilities $\{\overline{p}_n\}_{n=0}^{\infty}$ for this system.

Problem 7. In **Gambler's Ruin** two players engage in a game of chance in which A wins a dollar from B with probability p and B wins a dollar from A with probability $q = 1 - p$. There are N dollars between A and B and A begins the n dollars. They continue to play the game until A or B has won all of the money. What is the probability that A will end up with all the money assuming that $p > q$. Assume that $p = \frac{20}{38}$ which happens to be the house advantage in roulette. What is the probability that A will win all the money if $n = \$100$ and $N = \$1,000,000,000$?

Problem 8. Suppose that Urn I is chosen with probability $\frac{1}{2}$ and Urn II is also chosen with probability $\frac{1}{2}$. Suppose that Urn I has 5 white balls and 7 black balls and Urn II has 8 white balls and 3 black balls. After one of the urns is chosen, a ball is chosen at random from the urn. What is the probability that the urn was Urn I given that the ball chosen was white?

Problem 9. A test for a disease is positive with probability .95 when administered to a person with the disease. It is positive with probability .03 when administered to a person not having the disease. Suppose that the disease occurs in one in a million persons. Suppose that the test is administered to a person at random and the test is positive. What is the probability that the person has the disease?

Problem 10. Determine the natural cubic spline through the points $\{(0,-1),(1,0),(2,2),(3,0),(4,-2)\}$. Give the cubic polynomial for the spline on each of the intervals $\{[0,1],[1,2],[2,3],[3,4]\}$.