FALL 2019 QUIZ 1

JAMES KEESLING

The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on the tests.

1. Solving Equations

Problem 1. Suppose that $f : R \to R$ is continuous and suppose that for $a < b \in R$, $f(a) \cdot f(b) < 0$. Show that there is a c with $a < c < b$ such that $f(c) = 0$.

Problem 2. Solve the equation $x^5 - 3x^4 + 2x^3 - x^2 + x = 3$. Solve using the Bisection method. Solve using the Newton-Raphson method. How many solutions are there?

Problem 3. Solve the equation $x = \cos x$ by the Bisection method and by the Newton-Raphson method. How many solutions are there? Solve the equation $\sin(x) = \cos x$ by the Bisection method and by the Newton-Raphson method. How many solutions are there?

Problem 4. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 5. Solve the equation $x^4 = 2$ by the Newton-Raphson method. How many real solutions are there? For which starting values x_0 will the method converge?

Problem 6. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and that $f(z) = 0$. Suppose that $f'(z) \neq 0$. Let $g(x) = x - \frac{f(x)}{f'(x)}$. Show that there is an $\varepsilon > 0$ such that for any $x_0 \in [z - \varepsilon, z + \varepsilon]$, $g^n(x_0) \to z$ as $n \to \infty$

Problem 7. Show that the Newton-Raphson method converges quadratically. That is, suppose that the fixed point is z and that the error of the nth iteration is $|x_n - z| = h$, then $|x_{n+1} - z| \approx h^2$ for h small enough.