# MAD 4401 - JAMES KEESLING - QUIZ 2

**Problem 1.** Consider the following differential equation.

$$\frac{dx}{dt} = t \cdot x$$
$$x(0) = 1$$

Solve on the interval $[0, 1]$ using $h = .1$. Solve using the Taylor Method of degree 4, 5, 6, 7, and 8. Compare these results with Runge-Kutta using the same $h$.

**Problem 2.** Estimate $\frac{d^n f}{dx^n}$ at $x = a$ using the points $\{a - 4 \cdot h, a - 2 \cdot h, a - h, a, a + h, a + 2 \cdot h, a + 4 \cdot h\}$. For which $n$ can this be done? What is the best $h$? What is the error?

**Problem 3.** Compare Euler, Heun, and Runge-Kutta on $[0, 1]$ using $h = .1$.

$$\frac{dx}{dt} = t \cdot x$$
$$x(0) = 1$$

**Problem 4.** Use the Euler method to solve the following differential equation

$$\frac{dx}{dt} = x$$
$$x(0) = 1$$

Solve on $[0, 1]$ using $h = \frac{1}{n}$. Do this by hand to show that $x_i = \left(1 + \frac{1}{n}\right)^i$. What does this say about the following limit?

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

**Problem 5.** Solve $\frac{dx}{dt} = M \cdot x$ with $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

**Problem 6.** Solve $\frac{dx}{dt} = M \cdot x$ with $M = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ and with $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

**Problem 7.** Convert $\frac{d^2x}{dt^2} + x = 0$ to a first-order differential equation.

**Problem 8.** Convert $\frac{d^3x}{dt^3} + x = 0$ to a first-order differential equation.