FALL 2019 QUIZ 2

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The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on the tests.

1. Lagrange Polynomials

Problem 11. Determine the polynomial $p(x)$ of degree 5 passing through the points $\left\{(0,0),\left(\frac{1}{2},0\right),(1,0),\left(\frac{3}{2},1\right),(2,0),\left(\frac{5}{2},0\right)\right\}$. Determine the polynomials $L_i(x)$ for this set of x_i's where

$$L_i(x_j) = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

Problem 12. Determine the VanderMonde matrix for the points $\left[0, \frac{1}{9}, \frac{2}{9}, \ldots, 1\right]$.

2. Numerical Integration

Problem 13. Determine the closed Newton-Cotes coefficients for eleven points, $\{a_0, a_1, \ldots, a_{10}\}$. Use these values to estimate the integral

$$\int_{-4}^{4} \frac{1}{1+x^2} dx.$$

Problem 14. Suppose that $\{x_i\}_{i=0}^n$ is a set of points in R such that $x_i \neq x_j$ for all $i \neq j$. Let $j_0 \in \{0, 1, \ldots, n\}$. Give a formula for a polynomial $p(x)$ such that $p(x)$ has degree n and such that $p(x_j) = 0$ for $j \neq j_0$ and $p(x_{j_0}) = 1$.

Problem 17. Explain the Romberg method for approximating the integral. If the interval is divided into 2^n subintervals and the Romberg method is applied, what is the error of the method?