The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on the tests.

1. Lagrange Polynomials

**Problem 11.** Determine the polynomial $p(x)$ of degree 5 passing through the points \((0, 0), (\frac{1}{2}, 0), (1, 0), (\frac{3}{2}, 1), (2, 0), (\frac{5}{2}, 0)\). Determine the polynomials $L_i(x)$ for this set of $x_i$'s where

$$L_i(x_j) = \begin{cases} 
0 & i \neq j \\
1 & i = j
\end{cases}$$

**Problem 12.** Determine the VanderMonde matrix for the points \([0, \frac{1}{9}, \frac{2}{9}, \ldots, 1]\).

2. Numerical Integration

**Problem 13.** Determine the closed Newton-Cotes coefficients for eleven points, \(\{a_0, a_1, \ldots, a_{10}\}\). Use these values to estimate the integral

$$\int_{-4}^{4} \frac{1}{1 + x^2} \, dx.$$ 

**Problem 14.** Suppose that \(\{x_i\}_{i=0}^{n}\) is a set of points in $R$ such that $x_i \neq x_j$ for all $i \neq j$. Let $j_0 \in \{0, 1, \ldots, n\}$. Give a formula for a polynomial $p(x)$ such that $p(x)$ has degree $n$ and such that $p(x_j) = 0$ for $j \neq j_0$ and $p(x_{j_0}) = 1$.

**Problem 17.** Explain the Romberg method for approximating the integral. If the interval is divided into $2^n$ subintervals and the Romberg method is applied, what is the error of the method?