MAD 4401 QUIZ 4 SPRING 2018 - JAMES KEESLING

Problem 1. Solve the differential equation for $\frac{dx}{dt} = f(t,x) = t \cdot x^2$ with $x(0) = 1$. Solve using Picard iteration for four iterations.

Problem 2. Solve the differential equation for $\frac{dx}{dt} = f(t,x) = t \cdot x^2$ with $x(0) = 1$. Solve using the Euler method, Heun, and Runge-Kutta methods using $h = \frac{1}{20}$ and $n = 20$. Compare the answers and the errors for each of these methods.

Problem 3. Solve the differential equation for $\frac{dx}{dt} = f(t,x) = t \cdot x^2$ with $x(0) = 1$. Solve using Picard iteration for four iterations. Solve using the Taylor method of order 3,4, and 5. Compare the answers and the errors for each of these.

Problem 4. How would you go about solving the differential equation $\frac{d^2x}{dt^2} = -x$ with $x(0) = 1$ and $x'(0) = 1$ with each of the methods listed in the previous problem. What changes would need to be made in the programs? Solve this problem as a linear differential equation using the **linearode** program. Solve on the interval $[0,1]$ with $h = \frac{1}{10}$.

Problem 5. Find a Taylor expansion for the solution $x(t) = a_0 + a_1 t + a_2 t^2 + \cdots$ for the differential equation $\frac{dx}{dt} = t \cdot x$ with the boundary condition $x(0) = 1$. Solve for $\{a_0, a_1, a_2, a_3, a_4, a_5\}$. Do this by hand solving for these coefficients recursively. Solve for the coefficients using the Taylor Method program included in your program collection. Can you determine the general a_n?

Problem 6. Consider the following differential equation.

$$\frac{dx}{dt} = t \cdot x$$
$$x(0) = 1$$

Solve on the interval $[0,1]$ using $h = \frac{1}{10}$. Solve using the Taylor Method of degree 4. Compare the result with Runge-Kutta using the same h.

Problem 7. Use the Euler method to solve the following differential equation

$$\frac{dx}{dt} = x$$
$$x(0) = 1$$

Solve on $[0,1]$ using $h = \frac{1}{n}$. Do this by hand to show that $x_i = \left(1 + \frac{1}{n}\right)^i$. What does this say about the following limit?

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Problem 8. Solve $\frac{dx}{dt} = M \cdot x$ with $M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and with $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Problem 9. Convert $\frac{d^2x}{dt^2} + x = 0$ to a first-order differential equation. Solve over the interval $[0, \pi]$ with $h = \frac{\pi}{10}$ assuming the initial conditions $x(0) = 1$ and $x'(0) = 0$. Use the program **linearode**.

Problem 10. Convert $\frac{d^3x}{dt^3} + x = 0$ to a first-order differential equation. Solve this equation over the interval $[0, 1]$ for the initial conditions $x''(0) = 0$, $x'(0) = 1$, and $x(0) = 0$. Use the program **linearode**.