**Problem 1.** Solve the differential equation for \( \frac{dx}{dt} = f(t, x) = t \cdot x^2 \) with \( x(0) = 1 \). Solve using Picard iteration for four iterations.

**Problem 2.** Solve the differential equation for \( \frac{dx}{dt} = f(t, x) = t \cdot x^2 \) with \( x(0) = 1 \). Solve using the Euler method, Heun, and Runge-Kutta methods using \( h = \frac{1}{20} \) and \( n = 20 \). Compare the answers and the errors for each of these methods.

**Problem 3.** Solve the differential equation for \( \frac{dx}{dt} = f(t, x) = t \cdot x^2 \) with \( x(0) = 1 \). Solve using Picard iteration for four iterations. Solve using the Taylor method of order 3, 4, and 5. Compare the answers and the errors for each of these.

**Problem 4.** How would you go about solving the differential equation \( \frac{d^2 x}{dt^2} = -x \) with \( x(0) = 1 \) and \( x'(0) = 1 \) with each of the methods listed in the previous problem. What changes would need to be made in the programs? Solve this problem as a linear differential equation using the `linearode` program. Solve on the interval \([0, 1]\) with \( h = \frac{1}{10} \).

**Problem 5.** Find a Taylor expansion for the solution \( x(t) = a_0 + a_1 t + a_2 t^2 + \cdots \) for the differential equation \( \frac{dx}{dt} = t \cdot x \) with the boundary condition \( x(0) = 1 \). Solve for \( \{a_0, a_1, a_2, a_3, a_4, a_5\} \). Do this by hand solving for these coefficients recursively. Solve for the coefficients using the Taylor Method program included in your program collection. Can you determine the general \( a_n \)?

**Problem 6.** Consider the following differential equation.

\[
\frac{dx}{dt} = t \cdot x \\
x(0) = 1
\]

Solve on the interval \([0, 1]\) using \( h = \frac{1}{10} \). Solve using the Taylor Method of degree 4. Compare the result with Runge-Kutta using the same \( h \).

**Problem 7.** Use the Euler method to solve the following differential equation

\[
\frac{dx}{dt} = x \\
x(0) = 1
\]

Solve on \([0, 1]\) using \( h = \frac{1}{n} \). Do this by hand to show that \( x_i = \left(1 + \frac{1}{n}\right)^i \). What does this say about the following limit?
\[
\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n
\]

Problem 8. Solve \( \frac{dx}{dt} = M \cdot x \) with \( M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \) and with \( x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \).

Problem 9. Convert \( \frac{d^2x}{dt^2} + x = 0 \) to a first-order differential equation. Solve over the interval \([0, \pi]\) with \( h = \frac{\pi}{10} \) assuming the initial conditions \( x(0) = 1 \) and \( x'(0) = 0 \). Use the program \texttt{linearode}.

Problem 10. Convert \( \frac{d^3x}{dt^3} + x = 0 \) to a first-order differential equation. Solve this equation over the interval \([0, 1]\) for the initial conditions \( x''(0) = 0, \; x'(0) = 1, \; \text{and} \; x(0) = 0 \). Use the program \texttt{linearode}.