FALL 2019 QUIZ 7

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The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on the tests.

1. Differential Equations

Problem 25. Solve the differential equation for $\frac{dx}{dt} = f(t,x) = t \cdot x^2$ with $x(0) = 1$. Solve using Picard iteration for five iterations. Solve using the Taylor method of order 3,4, and 5. Solve using the Euler method, modified Euler, Heun, and Runge-Kutta methods using $h = \frac{1}{20}$ and $n = 20$. Compare the answers and the errors for each of these methods.

Problem 26. How would you go about solving the differential equation $\frac{d^2x}{dt^2} = -x$ with $x(0) = 1$ and $x'(0) = 1$ with each of the methods listed in the previous problem. What changes would need to be made in the programs? Solve this problem as a linear differential equation using the **linearode** program. Solve on the interval $[0,1]$ with $h = \frac{1}{10}$.

Problem 27. Find a Taylor expansion for the solution $x(t) = a_0 + a_1 t + a_2 t^2 + \cdots$ for the differential equation $\frac{dx}{dt} = t \cdot x$ with the boundary condition $x(0) = 1$. Solve for $\{a_0, a_1, a_2, a_3, a_4, a_5\}$. Do this by hand solving for these coefficients recursively. Solve for the coefficients using the Taylor Method program included in your program collection. Can you determine the general a_n?

Problem 28. Consider the following differential equation.

$$\frac{dx}{dt} = t \cdot x$$
$$x(0) = 1$$

Solve on the interval $[0,1]$ using $h = .1$. Solve using the Taylor Method of degree 4, 5, 6, 7, and 8. Compare these results with Runge-Kutta using the same h.

Problem 29. Compare Euler, Heun, and Runge-Kutta on $[0,1]$ using $h = .1$.

$$\frac{dx}{dt} = t \cdot x$$
$$x(0) = 1$$

Problem 30. Use the Euler method to solve the following differential equation

$$\frac{dx}{dt} = x$$
$$x(0) = 1$$

Solve on $[0, 1]$ using $h = \frac{1}{n}$. Do this by hand to show that $x_i = \left(1 + \frac{1}{n}\right)^i$. What does this say about the following limit?

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$$

Problem 31. Solve $\frac{dx}{dt} = M \cdot x$ with $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Problem 32. Solve $\frac{dx}{dt} = M \cdot x$ with $M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and with $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Problem 33. Convert $\frac{d^2x}{dt^2} + x = 0$ to a first-order differential equation. Solve over the interval $[0, \pi]$ with $h = \frac{\pi}{10}$ assuming the initial conditions $x(0) = 1$ and $x'(0) = 0$. Use the program **linearode**.

Problem 34. Convert $\frac{d^3x}{dt^3} + x = 0$ to a first-order differential equation. Solve this equation over the interval $[0, 1]$ for the initial conditions $x''(0) = 0$, $x'(0) = 1$, and $x(0) = 0$. Use the program **linearode**.