The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on the tests.

1. Differential Equations

**Problem 1.** Solve the following system of differential equations. Find the Taylor series of the solutions to ten terms. Use the initial conditions \(x_1(0) = 1\), \(x_2(0) = 0\), and \(x_3(0) = 0\).

\[
\begin{align*}
\frac{dx_1}{dt} &= \sin(x_2) \cdot x_3 \\
\frac{dx_2}{dt} &= x_1 \\
\frac{dx_3}{dt} &= x_2
\end{align*}
\]

**Problem 2.** Solve the following system of differential equations using the `linearode` program. Solve the numerical part using \(h = 1/10\) and \(n = 10\). Solve the Taylor series part up to degree ten. Use the initial conditions \(x_1(0) = 0\), \(x_2(0) = 1\), and \(x_3(0) = 2\).

\[
\begin{align*}
\frac{dx_1}{dt} &= 2x_1 + 3x_2 - x_3 \\
\frac{dx_2}{dt} &= x_3 \\
\frac{dx_3}{dt} &= x_1 - x_2
\end{align*}
\]

2. Stochastic Simulation

**Problem 3.** You toss 10 coins simultaneously. What is the probability that you get precisely 5 heads? If you simulated the toss of a coin ten times by `coin(10)` and did this 20 times, what is the probability of getting exact five heads four times?

**Problem 4.** There are three urns. In urn 1 there are three white balls and two black balls. In urn 2 there are five white balls and 1 black ball. In urn 3 there are two white balls and seven black balls. An urn is chosen at random and a ball chosen at random from the urn. Given that the ball chosen was white, what is the probability that it was chosen from urn 1?

**Problem 5.** A certain medical test is devised for a certain disease. Suppose that if a person has the disease and the test is applied, then the result is positive with probability .85. Suppose that if a person does not have the disease, the probability that the test is negative is .75. The probability of the disease in the general population is \(\frac{1}{500}\). If the test
is applied to a random person and the test is positive, what is the probability that the person has the disease?

**Problem 6.** Two people, $A$ and $B$, are playing roulette. Each time, the winner takes $1 from the loser. Suppose that $A$ has the house advantage. If $A$ has $100 and $B$ has $900 and they play until one person has all the money, what is the probability that $A$ will win all the money?