FALL 2019 QUIZ 8

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The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on the tests.

1. Differential Equations

Problem 1. Solve the following system of differential equations. Find the Taylor series of the solutions to ten terms. Use the initial conditions $x_1(0) = 1$, $x_2(0) = 0$, and $x_3(0) = 0$.

$$\begin{aligned} \frac{dx_1}{dt} &= \sin(x_2) \cdot x_3 \\ \frac{dx_2}{dt} &= x_1 \\ \frac{dx_3}{dt} &= x_2 \end{aligned}$$

Problem 2. Solve the following system of differential equations using the **linearode** program. Solve the numerical part using $h = 1/10$ and $n = 10$. Solve the Taylor series part up to degree ten. Use the initial conditions $x_1(0) = 0$, $x_2(0) = 1$, and $x_3(0) = 2$.

$$\begin{aligned} \frac{dx_1}{dt} &= 2x_1 + 3x_2 - x_3 \\ \frac{dx_2}{dt} &= x_3 \\ \frac{dx_3}{dt} &= x_1 - x_2 \end{aligned}$$

2. Stochastic Simulation

Problem 3. You toss 10 coins simultaneously. What is the probability that you get precisely 5 heads? If you simulated the ross of a coin ten times by **coin**(10) and did this 20 times, what is the probability of getting exact five heads four times?

Problem 4. There are three urns. In urn 1 there are three white balls and two black balls. In urn 2 there are five white balls and 1 black ball. In urn 3 there are two white balls and seven black balls. An urn is chosen at random and a ball chosen at random from the urn. Given that the ball chosen was white, what is the probability that it was chosen from urn 1?

Problem 5. A certain medical test is devised for a certain disease. Suppose that if a person has the disease and the test is applied, then the result is positive with probability .85. Suppose that if a person does not have the disease, the probability that the test is negative is .75. The probability of the disease in the general population is $\frac{1}{500}$. If the test is applied to a random person and the test is positive, what is the probability that the person has the disease?

Problem 6. Two people, A and B, are playing roulette. Each time, the winner takes \$1 from the loser. Suppose that A has the house advantage. If A has \$100 and B has \$900 and they play until one person has all the money, what is the probability that A will win all the money?