

## FALL 2019 QUIZ 9

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The problems that follow illustrate the methods covered in class. They are typical of the types of problems that will be on the tests.

### 1. STOCHASTIC SIMULATION

**Problem 1.** Give a calculator function that produces a random number from the probability density function  $\lambda \cdot \exp(-\lambda t)$ . How would the function change if you wanted to generate 100 independent such random numbers.

**Problem 2.** Consider a process that is generated by waiting for the  $k^{\text{th}}$  occurrence from a Poisson process with rate  $\lambda$ . Such a waiting time is called **Erlang**. Write a program to generate 100 such waiting times

**Problem 3.** Consider a the pdf given by  $\frac{1}{\pi \cdot (1+x^2)}$ . Write a program that generates independent random numbers from this pdf.

**Problem 4.** Consider a queue of the form M/M/ $\infty$ . Suppose that the arrivals are Poisson with rate  $\alpha$ . Suppose that the servers serve at a rate  $\sigma$ . Assume that the service times are random with exponential waiting time. Determine the average number of individuals being served in this system. What is the distribution of the number of individuals? How can this be used as a model for the erythrocytes in the bloodstream of the human body?

**Problem 5.** Consider a queue of the form M/M/1. Suppose that the arrivals are Poisson with rate  $\alpha$ . Suppose that the servers serve at a rate  $\sigma$ . Assume that the service times are random with exponential waiting time. Determine the average number of individuals either waiting to be served or being served. What is the probability,  $p_n$ , of  $n$  individuals being in the system? Of course,  $n = 0, 1, 2, \dots$ . What is the average number of individuals in the system?