

MAD 4401 TEST 1 – JAMES KEESLING

NAME _____

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Solve the equation $x^5 + 3x - 5 = 0$ by the Newton-Raphson method. Write down the Newton function and give the results of each iteration to twelve digits. Determine the solution to twelve digits. Circle the final answer.

Problem 2. Determine the points in $[-1, 1]$ and coefficients used in Gaussian Quadrature for 8 points. Estimate the integral $\int_2^7 \sin(x^2) dx$ using Gaussian Quadrature with 8 points. Give the estimate with twelve digits.

Problem 3. Give the polynomial $p(x)$ of degree 6 passing through the points

$$\left\{ (0, 1), \left(\frac{1}{2}, 2\right), (1, 0), \left(\frac{3}{2}, -1\right), \left(2, \frac{1}{2}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{7}{2}, \frac{5}{4}\right) \right\}.$$

Problem 4. Give the normalized Newton-Cotes coefficients used for 8 equal subdivisions of the interval (i.e., using 9 points). Using this number of equally spaced points, estimate $\int_{-1}^5 \cos(x^2) dx$.

Problem 5. Solve the equation $x^5 = 2$ using the Bisection Method. Come up with a starting interval $[a, b]$ in which you can guarantee a solution. Determine the number of iterations necessary so that the error is less than 10^{-6} . Give the answer to this accuracy.