MAD 4401 TEST 2 – JAMES KEESLING

NAME ____

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Determine the coefficients $\{A_0, A_1, A_2, A_3, A_4\}$ to estimate the following derivative at the given point.

$$\frac{d^2}{dx^2} f(x)\Big|_{a=1} \approx A_0 f(a-4h) + A_1 f(a-h) + A_2 f(a) + A_3 f(a+h) + A_4 f(a+5h)$$

where $f(x) = \sin(x^3)$. What is the best h to use in the estimate? Write down your best estimate. What error do you expect in your answer? Explain.

Problem 2. Give the solution of the linear differential equation given below.

$$\frac{dx}{dt} = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & -3 \\ 3 & 2 & 1 \end{bmatrix} x \qquad x(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Calculate the numerical value of the solution at $t = 3$?

Problem 3. Assume a queueing system with Poisson arrival rate of α and a single server with an exponential service rate σ. Assume that $\sigma > \alpha > 0$. This is an $M/M/1/FIFO$ queue. Determine the steady-state probabilities for n, $\{\overline{p}_n\}_{n=0}^{\infty}$ for this system.

Problem 4. Solve the differential equation numerically using Runge-Kutta over the interval $[0, 2]$ using $h = \frac{1}{3}$ and $n = 6$.

$$\frac{dx}{dt} = \sin(t^2 x^2) \qquad x(0) = 1$$

At each $t_i = 0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2$ give the value of the estimate x_i to 7 digits.

Problem 5. A test for a disease is positive with probability .98 when administered to a person with the disease. It is positive with probability .05 when administered to a person not having the disease. Suppose that the disease occurs in one out of ten thousand persons. Suppose that the test is administered to a person at random and the test is positive. What is the probability that the person has the disease. Solve this exactly using Bayes' Theorem. Use Monte-Carlo in the program **medicaltest** in your calculator library to simulate administering this test to 1000 random individuals. Give the number that tested positive in the simulation and the number who tested positive who actually had the disease.