MAD 4401 TEST 1 - JAMES KEESLING

NAME ______________________________

Do all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Solve the equation $x^7 + 2 = \tan x$ by the Newton-Raphson method. Give the Newton function. Find a starting point for which the method converges. Give the starting point and the iterations with five digits accuracy. Give the final answer to twelve digits and circle the final answer.

Problem 2. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 3. Give the Legendre polynomial of degree 7. Determine the points and weights of for Gaussian quadrature for 7 points in the interval $[-1, 1]$.

Problem 4. Estimate $\int_0^2 \exp(\sqrt{x})dx$ using Romberg Integration using 2^7 subintervals. Give the first column of the result to 5 digits and the last two columns to 12 digits. Circle the best answer.

Problem 5. Consider the points $\{\frac{1}{6}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{5}{6}\}$. What must the coefficients, $\{A_0, A_1, A_2, A_3, A_4\}$, be so that

$$\int_0^1 p(x)dx = A_0 \cdot p\left(\frac{1}{6}\right) + A_1 \cdot p\left(\frac{1}{4}\right) + A_2 \cdot p\left(\frac{1}{2}\right) + A_3 \cdot p\left(\frac{3}{4}\right) + A_4 \cdot p\left(\frac{5}{6}\right)$$

is exact for polynomials $p(x)$ of degree ≤ 4?