# MAD 4401 TEST 1 FALL 2017 - JAMES KEESLING

**Name:** _____

Do all problems and show all work. Each problem worth 20 points. Partial credit will be given for correct work even if the answer is wrong. Credit will be deducted for incorrect work even though the final answer may be correct.

**Problem 1.** Solve the equation $x^3 + 4 = \cos x^2$ by the Newton-Raphson method. Give the formula for the Newton function. Find a starting point for which the method converges. Give the starting point and the iterations with twelve digits accuracy. Circle the final answer.

**Problem 2.** Let $h$ be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

**Problem 3.** Give the Legendre polynomial of degree 8. Determine the points and weights of for Gaussian quadrature for 8 points in the interval $[-1, 1]$. Estimate the integral $\int_0^1 \frac{1}{1+x^2} dx$ using Gaussian quadrature with 8 points.

**Problem 4.** Estimate $\int_0^2 \cos(x^2) dx$ using Romberg Integration using $2^5$ subintervals. Give the first column of the result to 5 digits and the last two columns to 12 digits. Circle the best answer and say how many digits you feel are correct.

**Problem 5.** Determine the normalized Newton-Cotes coefficients using 5 subintervals. What points and what coefficients should you use to estimate the following integral by Newton-Cotes with five intervals $\int_{-2}^{5} f(x)dx$. Give the estimate.