MAD 4401 TEST 1 FALL 2019 – JAMES KEESLING

NAME _____

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Solve the equation $x^7 + 5x^6 + 3x - 5 = 0$ by the Newton-Raphson method. Write down the Newton function and give the results of each iteration to twelve digits. Determine the solution to twelve digits. Circle the final answer.

Problem 2. Determine the points in $[-1, 1]$ and coefficients used in Gaussian Quadrature for 8 points. Estimate the integral $\int_2^7 \sin(x^2)dx$ using Gaussian Quadrature with 8 points and with 15 points. Give these estimates with twelve digits.

Problem 3. Give the polynomial $p(x)$ of degree 7 passing through the points

$$\left\{(0,1), \left(\frac{1}{2}, 2\right), (1,0), \left(\frac{3}{2}, -1\right), \left(2, \frac{1}{2}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{7}{2}, \frac{5}{4}\right), (4,0)\right\}.$$

Problem 4. Give the normalized Newton-Cotes coefficients used for 8 equal subdivisions of the interval (i.e., using 9 points). Using this number of equally spaced points, estimate $\int_{-1}^{5} \cos(x^2) dx$.

Problem 5. Estimate the integral $\int_{-4}^{4} \frac{1}{1+x^2} dx$ using Romberg integration with $n = 7$. What is the best answer. What is your best estimate of the error of the answer?