MAD 4401 TEST 2 - JAMES KEESLING

NAME ______________________________

Do all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Determine the coefficients to compute the second derivative of $f(x) = \sin(x^3)$ at $a = 1$ using the points $\{a-3h, a-h, a, a+h, a+3h\}$. Give the estimate of the derivative as a function of h. Determine the best value of h for the greatest accuracy of the answer. How many digits accuracy can you expect with this choice of h?

Problem 2. Assume a queueing system with Poisson arrival rate of α and a single server with an exponential service rate σ. Assume that $\sigma > \alpha > 0$. This is an M/M/1/FIFO queue. Determine the steady-state probabilities for n, $\{\overline{p}_n\}_{n=0}^{\infty}$ for this system.

Problem 3. A test for a disease is positive with probability .96 when administered to a person with the disease. It is positive with probability .04 when administered to a person not having the disease. Suppose that the disease occurs in 3 out of 200,000 people. Suppose that the test is administered to a person at random and the test is positive. What is the probability that the person has the disease?

Problem 4. Determine the natural cubic spline through the points $\{(0,-2),(1,0),(2,2),(3,0)\}$. Give the cubic polynomial for the spline on each of the intervals $\{[0,1],[1,2],[2,3]\}$.

Problem 5. Solve the differential equation $\frac{dx}{dt} = f(t,x) = t^2 \cdot x$ with $x(0) = 1$. Solve using Picard iteration for five iterations. Solve using the Taylor method of order 5 on the interval $[0,1]$ with $h = \frac{1}{10}$ and $n = 10$.