MAD 4401 - TEST 2 - JAMES KEESLING

NAME ____

Work all problems and show all work. Partial credit will be given for logical analysis even if the final answer is incorrect. Credit will be deducted for work that is incorrect even if the final answer is correct.

Problem 1. Estimate $\frac{d^2 f}{dx^2}$ at $x = 1$ using the points $\{a - 6 \cdot h, a - 2 \cdot h, a - h, a, a + h, a + 2 \cdot h, a + 4 \cdot h\}$. Assume that $f(x) = \sin(x^2)$ and $a = 1$. Assume that the accuracy of the calculation of $f(x)$ is 10^{-14}. What is the best h? Estimate the error. Circle your best estimate.

Problem 2. Estimate the solution of the differential equation $\frac{dx}{dt} = f(t, x) = t \cdot x$ with $x(0) = 1$ using five iterations of Picard iteration.

Problem 3. Find a Taylor expansion for the solution $x(t) = a_0 + a_1 t + a_2 t^2 + \cdots$ for the differential equation $\frac{dx}{dt} = \sin(t) \cdot x$ with the boundary condition $x(0) = 1$. Determine the following coefficients $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$.

Problem 4. Convert $\frac{d^2x}{dt^2} + x = 0$ to a first-order differential equation. Solve over the interval $[0, \pi]$ with $h = \frac{\pi}{2}$ and $n = 2$ assuming the initial conditions $x(0) = 1$ and $x'(0) = 0$. Use the program **linearode**.

Problem 5. Use the **Queue** program to simulate a queueing system for $M/M/1/FIFO$ with $\alpha = 7$ per hour and $\sigma = 8$ per hour for a time period of 100 hours. Simulate a queueing system for $M/M/2/FIFO$ with $\alpha = 7$ per hour and $\sigma = 8$ per hour for a time period of 100 hours. Record the results of the simulations. How do the results compare with the theoretical calculations for the $M/M/1/FIFO$ case?