MAD 4401 FALL 2019 TEST 2 - JAMES KEESLING

NAME ______________________________

Do all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Determine the coefficients $\{A_0, A_1, A_2, A_3, A_4\}$ and the function $p(h)$ to estimate the second derivative of $f(x) = \sin(x^3)$ at $x = 2$.

$$\left.\frac{d^2 f}{dx^2}\right|_{x=a} \approx p(h) = A_0 \cdot f(a-3h) + A_1 \cdot f(a-h) + A_2 \cdot f(a) + A_3 \cdot f(a+h) + A_4 \cdot f(a+5h)$$

Problem 2. A medical test has the property that if it is administered to a person with the disease, the test is positive with probability .91. If the person does not have the disease, the probability of a false positive is .12. If the disease has a probability of $\frac{1}{600}$ and it is administered to a random person and the test is positive, what is the probability that the person has the disease?

Problem 3. What is the probability of 6 events in an interval of length 2 for a Poisson process with rate $\lambda = 3$? Give the formula for the probability of n events in the interval and use this to calculate the probability for $n = 6$.

Problem 4. Assume that human blood contains 20×10^{12} erythrocytes. Suppose that each erythrocyte lives 90 days. Explain how to model this as an M/M/∞ queue. How many erythrocytes are being produced each day by the bone marrow?

Problem 5. Solve the following differential equation using **linearode**. Solve on $[0,1]$ using $h = 1/10$ and $n = 10$. Solve using the initial conditions $x(0) = -1$ and $y(0) = 0$. Solve for five terms in the Taylor expansion part of the program.

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = x$$