

MAD 4401 - TEST 2 - JAMES KEESLING

NAME \_\_\_\_\_

Work all problems and show all work. Partial credit will be given for logical analysis even if the final answer is incorrect. Credit will be deducted for work that is incorrect even if the final answer is correct.

**Problem 1.** Determine the cubic functions for the cubic spline going through the points  $\{(1, 0), (2, 1), (3, -1), (4, 3)\}$ .

$$S_0(x) = \qquad \qquad \qquad 1 \leq x < 2$$

$$S_1(x) = \qquad \qquad \qquad 2 \leq x < 3$$

$$S_2(x) = \qquad \qquad \qquad 3 \leq x \leq 4$$

**Problem 2.** Estimate the solution of the differential equation  $\frac{dx}{dt} = f(t, x) = t \cdot x^3$  with  $x(0) = 2$  using three iterations of Picard iteration.

**Problem 3.** Find a Taylor expansion for the solution  $x(t) = a_0 + a_1t + a_2t^2 + \dots$  for the differential equation  $\frac{dx}{dt} = \exp(t) \cdot x$  with the boundary condition  $x(0) = 1$ . Determine the following coefficients  $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$ .

**Problem 4.** Convert  $\frac{d^2x}{dt^2} + x = 0$  to a first-order differential equation. Solve over the interval  $[0, \pi]$  with  $h = \frac{\pi}{4}$  and  $n = 4$  assuming the initial conditions  $x(0) = 1$  and  $x'(0) = 0$ . Use the program **linearode**.

**Problem 5.** Which would be better for the customer: a queueing system with  $\alpha = 9$ ,  $\sigma = 10$ , with one server or two servers with  $\alpha = 9$  and  $\sigma = 6$ ? Use the **Queue** program to justify your answer and explain.