MAP 2302 FINAL - JAMES KEESLING

NAME _____

Do all problems. Each problem is worth 10 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Solve the following linear homogeneous differential equation.

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$$

Problem 2. Solve the following differential equation. Use either the method of undetermined coefficients or the method of variation of parameters. Give the complete solution

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = \cos(5t)$$

Problem 3. Show that $\sin(2t)$ and $\sin(t)$ are linearly independent. Give a homogeneous, linear differential equation that has both of these functions as solutions.

Problem 4. Convert the following differential equation into first order and solve by matrix methods. Give the numerical solution for $x''(0) = 2$, $x'(0) = -1$, and $x(0) = 1$ using $h = \frac{1}{2}$ and $n = 2$.

$$\frac{d^3x}{dt^3} - 2\frac{dx}{dt} + x = 0$$

Problem 5. Determine the Laplace transform of the solution $x(t)$ to the following differential equation. [Note: You need not determine the solution, only the Laplace transform of the solution.]

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4 = \sin(2t) \quad x(0) = 1 \quad x'(0) = -2$$

Problem 6. Do five iterations of *Picard iteration* for the following differential equation. Start with $x_0(t) \equiv 2$ and determine $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, and $x_5(t)$.

$$\frac{dx}{dt} = t \cdot x + x \qquad x(0) = 2$$

Problem 7. Solve the differential equation $(x^3 + y)\,dx + (y + x + 3)\,dy = 0$.

Problem 8. Solve the differential equation $x^2 dx + xy\,dy = 0$.

Problem 9. What is the pressure 20 feet below the surface of the ocean? What is the pressure two miles under the surface of the ocean? Assume that the density of seawater is 1029 kg/m^3?

Problem 10. Solve the following differential equations.

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$$

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 0$$

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = 0$$