MAP 2302 PRACTICE PROBLEMS FOR TEST 2 - JAMES KEESLING

NAME ____

Do all problems. For the five problems on the test, each problem is worth 20 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Consider a mass attached to a spring resting on a frictionless surface as in the accompanying diagram. Assume that the spring obeys Hooke's Law, that is, $F = -k \cdot x$ where k is Hooke's constant and x is the distance that the mass is from the equilibrium position for the spring. Determine the differential equation that governs the movement of the mass. Describe the possible solutions of the equation.

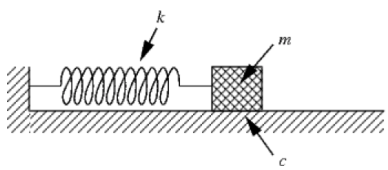

FIGURE 1. Mass-Spring Arrangement

Problem 2. Consider a homogeneous second order differential equation with constant coefficients. It can be put in the following form.

$$a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = 0 \tag{1}$$

The polynomial $p(x) = a_2 m^2 + a_1 m + a_0$ is called the *auxiliary polynomial.* Explain how the roots of $p(x)$ determine the solutions of (1). Describe the possibilities for the solution based on the roots.

Problem 3. Determine linear differential equations that have the following sets of functions as solutions. In which cases are all linear combinations of the functions the complete solution of the differential equation?

$$\{\sin(t), \cos(t)\} \quad \{\exp(t), \exp(2t)\} \quad \{\sin(t), \exp(t), \exp(2t)\} \quad \{\exp(t), 2t\exp(t)\}$$

Problem 4. Solve the following differential equation using Laplace transforms.

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = \exp(2t) \quad x(0) = 1 \quad x'(0) = -1$$