Do all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

**Problem 1.** Solve the following linear homogeneous differential equation.

\[
\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0
\]

**Problem 2.** Solve the following differential equation. Use either the method of undetermined coefficients or the method of variation of parameters.

\[
\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = \cos(5t)
\]
**Problem 3.** Show that \( \exp(t) \) and \( \cos(t) \) are linearly independent. Give a homogeneous, linear differential equation that has both of these functions as solutions.

**Problem 4.** Convert the following differential equation into first order and solve by matrix methods. Give the numerical solution for \( x''(0) = 2, x'(0) = -1, \) and \( x(0) = 1 \) using \( h = \frac{1}{2} \) and \( n = 2. \)

\[
\frac{d^3 x}{dt^3} - 2 \frac{dx}{dt} + x = 0
\]
Problem 5. Determine the Laplace transform of the solution \( x(t) \) to the following differential equation. [Note: You need not determine the solution, only the Laplace transform of the solution.]

\[
\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + x = \sin(2t) \quad x(0) = 1 \quad x'(0) = -2
\]