MTG 5316/4302 ASSIGNMENT 1

JAMES KEESLING

These problems are due on September 8, 2014. You may discuss the problems with members of the class and with me. You may consult our textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any sources.

Problem 1 (Problem 1.10.7 page 67 in Munkres) Let J be a well-ordered set. A subset J_0 is said to be **inductive** if for every $\alpha \in J$

$$(S_\alpha \subset J_0) \implies \alpha \in J_0.$$

Prove the following theorem.

Theorem (The principle of transfinite induction)**.** *If J is a well-ordered set and J_0 is an inductive subset of J, then $J_0 = J$.*

Problem 2 Suppose that X_α is non-empty for every $\alpha \in A$. Show that

$$\prod_{\alpha \in A} X_\alpha \neq \emptyset.$$

Problem 3 Suppose that $X_\alpha \neq \emptyset$ for all $\alpha \in A$. Let $\beta \in A$ and define

$$\pi_\beta : \prod_{\alpha \in A} X_\alpha \to X_\beta$$

by $\pi_\beta(x_\alpha) = x_\beta$. Show that π_β is onto for every $\beta \in A$.

Problem 4 Show that for every set X, there is no function $f : X \to 2^X$ such that f is onto.