These problems are due on September 8, 2014. You may discuss the problems with
members of the class and with me. You may consult our textbook and other books. You
may not read the papers of other students. The final writeup must be done by yourself in
your own words. It must not be copied from any sources.

Problem 1 (Problem 1.10.7 page 67 in Munkres) Let $J$ be a well-ordered set. A subset
$J_0$ is said to be inductive if for every $\alpha \in J$

\[(S_\alpha \subset J_0) \implies \alpha \in J_0.\]

Prove the following theorem.

Theorem (The principle of transfinite induction). If $J$ is a well-ordered set and $J_0$ is an
inductive subset of $J$, then $J_0 = J$.

Problem 2 Suppose that $X_\alpha$ is non-empty for every $\alpha \in A$. Show that

\[\prod_{\alpha \in A} X_\alpha \neq \emptyset.\]

Problem 3 Suppose that $X_\alpha \neq \emptyset$ for all $\alpha \in A$. Let $\beta \in A$ and define

\[\pi_\beta : \prod_{\alpha \in A} X_\alpha \to X_\beta\]

by $\pi_\beta(x_\alpha) = x_\beta$. Show that $\pi_\beta$ is onto for every $\beta \in A$.

Problem 4 Show that for every set $X$, there is no function $f : X \to 2^X$ such that $f$ is
onto.