

MTG 5316/4302 FALL 2018 ASSIGNMENT 2

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These problems are due in class on Friday, September 14, 2018.

You may discuss the problems with members of the class and with me. You may consult the textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any source.

Problem 1 State and prove the **Bolzano-Weierstrass Theorem** for the real line.

Problem 2 Define what it means to be **sequentially compact**. Show that a set $A \subset \mathbb{R}$ is sequentially compact if and only if A is closed and bounded.

Problem 3 Define a function from the Cantor Set onto the interval $[0, 1]$. Show that this function is continuous using the $\epsilon - \delta$ definition of continuity.

Problem 4 Define **uniform continuity**. Suppose that $f : X \rightarrow Y$ is continuous with X sequentially compact. Show that f is uniformly continuous.

Problem 5 Suppose that $f : X \rightarrow Y$ is continuous. Suppose that X is sequentially compact. Show that $f(X)$ is sequentially compact.