MTG 5316/4302 ASSIGNMENT 2

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These problems are due on September 15, 2014. You may discuss the problems with members of the class and with me. You may consult our textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any sources.

Problem 1. Let (X, d) be a metric space and let $(X, \mathcal{T})$ be the associated topological space. Show that U is an open set in $(X, \mathcal{T})$, i.e., $U \in \mathcal{T}$, if and only if whenever $\{x_i\}_{i=1}^{\infty} \subset X$ and $x_i \to z \in U$, then there is an N such that for all $i \geq N$, $x_i \in U$.

Problem 2. Let (X, d) be a metric space and let $(X, \mathcal{T})$ be the associated topological space. Show that $(X, \mathcal{T})$ is $\mathbf{T}_0$, $\mathbf{T}_1$, $\mathbf{T}_2$, $\mathbf{T}_3$, and $\mathbf{T}_4$.

Problem 3. Let (X, d) be a metric space and let $A, B \subset X$ be closed subsets of X such that $A \cap B = \emptyset$. Show that there is a continuous function $f : X \to [0, 1]$ such that $A \subset f^{-1}(1)$ and $B \subset f^{-1}(0)$.