MTG 5316/4302 FALL 2018 ASSIGNMENT 3

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These problems are due in class on Friday, September 21, 2018.

You may discuss the problems with members of the class and with me. You may consult the textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any source.

Problem 1 Let X be a metric space. Let $A \subset X$ be connected. State and prove the **Chain Connectedness Theorem** for A.

Problem 2 Let U be an open subset of $\mathbb{R}^n$. Show that if U is connected, then for every $a \neq b \in U$, there is a continuous $f : [0,1] \to U$ such that f is one-to-one with $f(0) = a$ and $f(1) = b$.

Problem 3 Let X be a metric space with $A \subset X$. The **closure** of A is the set $\overline{A} = \left\{ x \in X \;\middle|\; \exists \{x_i\}_{i=1}^{\infty} \subset A \ni \lim_{i \to \infty} x_i = x \right\}$. Note that for any set $A \subset X$, $\overline{A}$ is closed in X. Suppose that $A \subset X$ is connected. Show that $\overline{A}$ is connected.

Problem 4 Let X be a metric space and suppose that $A_\lambda \subset X$ is connected for all $\lambda \in \Lambda$. Suppose that there is an $x_0 \in X$ such that $x_0 \in A_\lambda$ for all λ. Show that

$$\bigcup_{\lambda \in \Lambda} A_\lambda$$

is connected.

Problem 5 Let $A \subset [0,1] \times [-1,1] \subset \mathbb{R}^2$ be defined by $A = \left\{ \left(x, \sin\left(\frac{1}{x}\right)\right) \;\middle|\; 0 < x \leq 1 \right\} \cup \{0\} \times [-1,1]$. This is the **Topologist's Sine Curve**. Show that A is connected. Show that there is no continuous function $f : [0,1] \to A$ such that $f(0) = (1, \sin(1))$ and $f(1) = (0,0)$.