MTG 5316/4302 FALL 2018 ASSIGNMENT 4

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These problems are due in class on Friday, September 28, 2018.

You may discuss the problems with members of the class and with me. You may consult the textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any source.

Problem 1 Let X be a metric space. Define **compactness** of X in terms of open covers of X. Show that X is compact if and only if for every collection of closed sets, $\mathscr{F} = \{F_\alpha\}_{\alpha \in A}$ if $\mathscr{F}$ has the finite intersection property, then

$$\bigcap_{\alpha \in A} F_\alpha \neq \emptyset.$$

Problem 2 Let X be a metric space. Show that X is sequentially compact if and only if X is compact.

Problem 3 Let X be a metric space. Suppose that $A \subset X$ be compact. Show that A is closed in X.

Problem 4 Let X be a compact metric space. Suppose that $f : X \to Y$ be continuous, one-to-one, and onto. Show that $f^{-1} : Y \to X$ is also continuous. In case f and f^{-1} are both continuous, f is said to be a **homeomorphism** between X and Y and X and Y are said to be **homeomorphic**.

Problem 5 Are the following pairs of spaces homeomorphic? (1) $[0,1]$ and S^1. (2) $[0,1]$ and $[0,1] \times [0,1]$. (3) $[0,1]$ and $[0,1] \cup [2,3]$. (4) (a,b) with $a < b$ and $\mathbb{R}$.