MTG 5316/4302 ASSIGNMENT 4

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These problems are due on October 3, 2014. You may discuss the problems with members of the class and with me. You may consult our textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any sources.

Problem 1. Suppose that X is a complete metric space and let $f : X \to X$ be a contraction mapping. Show that f has a unique fixed point in X. A *contraction mapping* is a function g from a metric space Z to itself such that there is a $0 < c < 1$ such that for all $x, y \in Z$, $d(g(x), g(y) \leq c \cdot d(x, y)$

Problem 2. Suppose that $h : X \to X$ is a continuous function on the metric space X. Suppose that for some $x_0 \in X$, $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 3. Use the Baire Category Theorem to show that the set of irrational points in $\mathbb{R}$ is dense in $\mathbb{R}$.