MTG 5316/4302 FALL 2018 ASSIGNMENT 5

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These problems are due in class on Friday, October 5, 2018.

You may discuss the problems with members of the class and with me. You may consult the textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any source.

Problem 1 Let X be a metric space. Suppose that $\{A_i\}_{i=1}^{\infty}$ is a countable collection of compact connected sets in X such that $A_{i+1} \subset A_i$ for all i. Show that

$$\bigcap_{i=1}^{\infty} A_i$$

is compact and connected.

Problem 2 Show that the **dyadic solenoid** is compact and connected.

Problem 3 Let $D = \prod_{i=1}^{\infty}\{0,1\}$. Give D the metric $d\left((a_i),(b_i)\right) = \sum_{i=1}^{\infty} \frac{|a_i - b_i|}{2^i}$. Show that D is homeomorphic to the Cantor middle third set.

Problem 4 Let D be as in Problem 3. Let $\mathbb{N} = \left\{(a_i) \in \prod_{i=1}^{\infty}\{0,1\} \;\middle|\; \exists N \; a_i = 0 \; \forall i > N\right\}$. Show that for the closure of $\mathbb{N}$, $\overline{\mathbb{N}}$ is all of D. If $A \subset X$ is such that $\overline{A} = X$, we say that A is **dense** in X.

Problem 5 Let $f : X \to Y$ be an onto function with X and Y metric spaces. We say that f is a **quotient map** provided that $U \subset Y$ is open if and only if $f^{-1}(U)$ is open in X. Show that if f is a quotient map, then f is continuous. Also show that if X is a compact metric space and f is continuous, then f is a quotient map.