MTG 5316/4302 FALL 2018 ASSIGNMENT 6

JAMES KEESLING

These problems are due in class on Friday, October 12, 2018.

You may discuss the problems with members of the class and with me. You may consult the textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any source.

Problem 1. Let X be a topological space. Suppose that $f : X \to Y$ is continuous. Show that if $A \subset X$ is connected, then $f(A) \subset Y$ is connected.

Problem 2. Suppose that X is a Hausdorff space. Show that if $A \subset X$ is compact, then A is closed in X.

Problem 3. Suppose that X and Y are compact Hausdorff spaces. Suppose that $f : X \to Y$ is continuous and onto. Show that f is a quotient map,

Problem 4. Suppose that X is a compact Hausdorff space. Suppose that $\{A_\lambda\}_{\lambda\in\Lambda}$ is a **chain** of closed connected sets. Show that

$$\bigcap_{\lambda\in\Lambda} A_\lambda$$

is a compact connected set. This collection being a chain means that for any λ_1 and λ_2, either $A_{\lambda_1} \subset A_{\lambda_2}$ or $A_{\lambda_2} \subset A_{\lambda_1}$.

Problem 5. Suppose that X is a Hausdorff space and that A and B are disjoint compact subsets of X. Show that there are disjoint sets U and V open in X such that $A \subset U$ and $B \subset V$.