

MTG 5316/4302 ASSIGNMENT 7

JAMES KEESLING

These problems are due on October 27, 2014. You may discuss the problems with members of the class and with me. You may consult our textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any sources.

Problem 1. Suppose that $U \subset \mathbb{R}^n$ is a connected open set. Show that for any pair of points $x, y \in U$, there is a continuous function $f : [0, 1] \rightarrow U$ such that $f(0) = x$ and $f(1) = y$.

Problem 2. The Tychonoff Theorem. Suppose that $\{X_\alpha\}_{\alpha \in A}$ is a collection of compact spaces. Show that $\prod_{\alpha \in A} X_\alpha$ is also compact.

Problem 3. Suppose that X is Hausdorff (\mathbf{T}_2). Show that if $\{A_i\}_{i=1}^\infty$ is a nested sequence of compact connected sets, then

$$\bigcap_{i=1}^{\infty} A_i$$

is compact and connected.