MTG 5316/4302 FALL 2018 ASSIGNMENT 8

JAMES KEESLING

These problems are due in class on Friday, October 26, 2018.

You may discuss the problems with members of the class and with me. You may consult the textbook and other books. You may not read the papers of other students. The final writeup must be done by yourself in your own words. It must not be copied from any source.

Problem 1. State **Urysohn's Lemma** for normal spaces. Prove Urysohn's Lemma for **metric** spaces.

Problem 2. State and prove the **Tietze Extension Theorem** for normal spaces. Assume Urysohn's Lemma.

Problem 3. A topological space X is **second countable** provided that there is a countable base, $\mathscr{B} = \{U_i\}_{i=1}^{\infty}$, for the topology of X. Assume that X is a normal Hausdorff space that is second countable. Show that X is metrizable.

Problem 4. A space X is **first countable** provided that for each point $x \in X$, there is a countable set of neighborhoods of x, $\mathscr{B}_x = \{U_i\}_{i=1}^{\infty}$, such that for any open U with $x \in U$, there exists a $U_i \in \mathscr{B}_x$ with $x \in U_i \subset U$. Give an example of a first countable space X that has a countable dense set $A \subset X$ such that X is not metrizable.

Problem 5. Give an example of a space X which is $\mathbb{T}_1$ but not Hausdorff.