MTG 5316/4302 FALL 2018 FINAL

JAMES KEESLING

Name: ______________________________________

1. Major Theorems and Proofs

In this section state and prove each theorem. The proofs should be complete without being tedious. Each problem is worth 10 points.

Problem 1. Suppose that X is a Hausdorff space. Show that if A and B are disjoint compact subsets of X, then there are disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

Problem 2. State and prove the **Contraction Mapping Theorem**.

Problem 3. Define **quotient map**. Let $f : X \to Y$ be an onto function with X and Y Hausdorff spaces. Show that if X is a compact space and f is continuous, then f is a quotient map.

Problem 4. Let X be a topological space. Suppose that $f : X \to Y$ is continuous. Show that if $A \subset X$ is connected, then $f(A) \subset Y$ is connected.

Problem 5. Let $\mathbb{R}_s$ be the **Sorgenfrey line**. Show that $\mathbb{R}_s$ is a normal space.

2. Examples and Minor Proofs

In this section, give brief examples, counterexamples, or quick proofs. Each problem is worth 5 points.

Problem 6. Suppose that $A \subset \mathbb{R}$ is connected. Show that A is an interval.

Problem 7. Is it possible to find a continuous function $f : [0,1] \to \{0,1\}$ which is onto where $\{0,1\}$ is the set of two points with the discrete topology?

Problem 8. State the **Bolzano-Weierstrass Theorem** for the real line..

Problem 9. Define a continuous function $f : C \to [0, 1]$ which is onto where C is the Cantor set and $[0, 1]$ is the unit interval.

Problem 10. Give an example of a space X which is $\mathbb{T}_1$ but not Hausdorff.

Problem 11. Give a brief proof that if X is any set, there is no function $f : X \to \mathscr{P}(X)$ which is onto. Here $\mathscr{P}(X)$ is the **power set** of X or the set of all subsets of X.

Problem 12. Let X be a topological space. Define the **cone** of X, $c(X)$.

Problem 13. State the **Baire Category Theorem**.

Problem 14. Define the **dyadic solenoid**. Show that the dyadic solenoid is compact and connected.

Problem 15. What does it mean for a space X to be **first countable**? Show that every metric space is first-countable.