MTG 5316/4302 FALL 2018 REVIEW FOR FINAL QUIZ 1

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Problem 1. Show that if a set $A \subset \mathbb{R}$ is connected, then it must be an interval.

Problem 2. Suppose that $f : X \to Y$ is continuous with X and Y metric spaces. Show that if $A \subset X$ is connected, then $f(A) \subset Y$ is connected.

Problem 3. Show that if $A \subset \mathbb{R}$ is an interval, then it is connected.

Problem 4. State and prove the **Bolzano-Weierstrass Theorem** for the real line.

Problem 5. Define what it means to be **sequentially compact**. Show that a set $A \subset \mathbb{R}$ is sequentially compact if and only if A is closed and bounded. The **Heine-Borel Theorem** states that $A \subset \mathbb{R}^n$ is compact if and only if it is closed and bounded.

Problem 6. Define a function from the Cantor Set onto the interval $[0, 1]$. Show that this function is continuous using the $\epsilon - \delta$ definition of continuituy.

Problem 7. Define **uniform continuity**. Suppose that $f : X \to Y$ is continuous with X sequentially compact. Show that f is uniformly continuous.

Problem 8. Suppose that $f : X \to Y$ is continuous. Suppose that X is sequentially compact. Show that $f(X)$ is sequentially compact.

Problem 9. Let X be a metric space. Let $A \subset X$ be connected. State and prove the **Chain Connectedness Theorem** for A.