Problem 1. Let $U$ be an open subset of $\mathbb{R}^n$. Show that if $U$ is connected, then for every $a \neq b \in U$, there is a continuous $f : [0, 1] \to U$ such that $f$ is one-to-one with $f(0) = a$ and $f(1) = b$.

Problem 2. Let $X$ be a metric space with $A \subset X$. The closure of $A$ is the set $\overline{A} = \{ x \in X \mid \exists \{ x_i \}_{i=1}^{\infty} \subset A \ni \lim_{i \to \infty} x_i = x \}$. Note that for any set $A \subset X$, $\overline{A}$ is closed in $X$. Suppose that $A \subset X$ is connected. Show that $\overline{A}$ is connected.

Problem 3. Let $X$ be a metric space and suppose that $A_\lambda \subset X$ is connected for all $\lambda \in \Lambda$. Suppose that there is an $x_0 \in X$ such that $x_0 \in A_\lambda$ for all $\lambda$. Show that

$$\bigcup_{\lambda \in \Lambda} A_\lambda$$

is connected.

Problem 4. Let $A \subset [0, 1] \times [-1, 1] \subset \mathbb{R}^2$ be defined by $A = \{(x, \sin(\frac{1}{x})) \mid 0 < x \leq 1\} \cup \{0\} \times [-1, 1]$. This is the Topologist’s Sine Curve. Show that $A$ is connected. Show that there is no continuous function $f : [0, 1] \to A$ such that $f(0) = (1, \sin(1))$ and $f(1) = (0, 0)$.

Problem 5. Let $X$ be a metric space. Define compactness of $X$ in terms of open covers of $X$. Show that $X$ is compact if and only if for every collection of closed sets, $\mathcal{F} = \{F_\alpha\}_{\alpha \in A}$ if $\mathcal{F}$ has the finite intersection property, then

$$\bigcap_{\alpha \in A} F_\alpha \neq \emptyset.$$

Problem 6. Let $X$ be a metric space. Show that $X$ is sequentially compact if and only if $X$ is compact.

Problem 7. Let $X$ be a compact metric space. Suppose that $f : X \to Y$ be continuous, one-to-one, and onto. Show that $f^{-1} : Y \to X$ is also continuous. In case $f$ and $f^{-1}$ are
both continuous, $f$ is said to be a **homeomorphism** between $X$ and $Y$ and $X$ and $Y$ are said to be **homeomorphic**.

**Problem 8.** Let $X$ be a metric space. Suppose that $\{A_i\}_{i=1}^{\infty}$ is a countable collection of compact connected sets in $X$ such that $A_{i+1} \subset A_i$ for all $i$. Show that

$$\bigcap_{i=1}^{\infty} A_i$$

is compact and connected.

**Problem 9.** Show that the dyadic solenoid is compact and connected.