MTG 5316/4302 FALL 2018 REVIEW FOR FINAL QUIZ 3

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Problem 1. Suppose that X is a Hausdorff space. Show that if $A \subset X$ is compact, then A is closed in X.

Problem 2. Suppose that X and Y are compact Hausdorff spaces. Suppose that $f : X \to Y$ is continuous and onto. Show that f is a quotient map,

Problem 3. Suppose that X is a compact Hausdorff space. Suppose that $\{A_\lambda\}_{\lambda \in \Lambda}$ is a **chain** of closed connected sets. Show that

$$\bigcap_{\lambda \in \Lambda} A_\lambda$$

is a compact connected set. This collection being a chain means that for any λ_1 and λ_2, either $A_{\lambda_1} \subset A_{\lambda_2}$ or $A_{\lambda_2} \subset A_{\lambda_1}$.

Problem 4. Suppose that X is a Hausdorff space and that A and B are disjoint compact subsets of X. Show that there are disjoint sets U and V open in X such that $A \subset U$ and $B \subset V$.

Problem 5. Let $\mathbb{R}_s$ be the **Sorgenfrey line**. Show that $\mathbb{R}_s$ is a normal space.

Problem 6. Let $X \subset \mathbb{R}_s$. Show that X is normal in the subspace topology.

Problem 7. Show that a compact subset of $\mathbb{R}_s$ is countable.

Problem 8. Show that the rationals $\mathbb{Q}$ are dense in $\mathbb{R}_s$.

Problem 9. Show that $\mathbb{Q} \times \mathbb{Q} \subset \mathbb{R}_s \times \mathbb{R}_s$ is dense. Show that $\mathbb{R}_s \times \mathbb{R}_s$ has an uncountable closed discrete subspace.