MTG 5316/4302 FALL 2018 REVIEW FOR FINAL QUIZ 3

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Problem 1. State **Urysohn's Lemma** for normal spaces. Prove Urysohn's Lemma for **metric** spaces.

Problem 2. State and prove the **Tietze Extension Theorem** for normal spaces. Assume Urysohn's Lemma.

Problem 3. A topological space X is **second countable** provided that there is a countable base, $\mathcal{B} = \{U_i\}_{i=1}^{\infty}$, for the topology of X. Assume that X is a normal Hausdorff space that is second countable. Show that X is metrizable.

Problem 4. A space X is **first countable** provided that for each point $x \in X$, there is a countable set of neighborhoods of x, $\mathcal{B}_x = \{U_i\}_{i=1}^{\infty}$, such that for any open U with $x \in U$, there exists a $U_i \in \mathcal{B}_x$ with $x \in U_i \subset U$. Give an example of a first countable space X that has a countable dense set $A \subset X$ such that X is not metrizable.

Problem 5. Give an example of a space X which is $\mathbb{T}_1$ but not Hausdorff.

Problem 6. State and prove the **Contraction Mapping Theorem**.

Problem 7. State and prove the **Baire Category Theorem**.

Problem 8. Let X be a topological space. Define the **cone** of X, $c(X)$. Show that if $X = \mathbb{N}$ with the discrete topology, then $c(\mathbb{N})$ is not metrizable.

Problem 9. Let X be a topological space. Define the **mapping torus** of X denoted by T_f. Let $X = \mathbb{S}^1$ and $f = id : \mathbb{S}^1 \to \mathbb{S}^1$. What is T_f in this case? Suppose that $f : \mathbb{S}^1 \to \mathbb{S}^1$ is rotation by π. What is the mapping torus, T_f, in this case?