

Problem 3. A topological space $X$ is second countable provided that there is a countable base, $\mathcal{B} = \{U_i\}_{i=1}^{\infty}$, for the topology of $X$. Assume that $X$ is a normal Hausdorff space that is second countable. Show that $X$ is metrizable.

Problem 4. A space $X$ is first countable provided that for each point $x \in X$, there is a countable set of neighborhoods of $x$, $\mathcal{B}_x = \{U_i\}_{i=1}^{\infty}$, such that for any open $U$ with $x \in U$, there exists a $U_i \in \mathcal{B}_x$ with $x \in U_i \subset U$. Give an example of a first countable space $X$ that has a countable dense set $A \subset X$ such that $X$ is not metrizable.

Problem 5. Give an example of a space $X$ which is $T_1$ but not Hausdorff.


Problem 7. State and prove the Baire Category Theorem.

Problem 8. Let $X$ be a topological space. Define the cone of $X$, $c(X)$. Show that if $X = \mathbb{N}$ with the discrete topology, then $c(\mathbb{N})$ is not metrizable.

Problem 9. Let $X$ be a topological space. Define the mapping torus of $X$ denoted by $T_f$. Let $X = S^1$ and $f = id : S^1 \to S^1$. What is $T_f$ in this case? Suppose that $f : S^1 \to S^1$ is rotation by $\pi$. What is the mapping torus, $T_f$, in this case?