MTG 5316/4302 PRACTICE FINAL EXAM

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Work all problems and show all work. Each problem is worth ten points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and calculations that are not correct.

Problem 1. Show that $K \subset \mathbb{R}$ is compact if and only if K is closed and bounded.

Problem 2. State and prove the **Contraction Mapping Theorem**.

Problem 3. Let X be a connected space and suppose that $f : X \to Y$ is continuous and onto. Show that Y is connected.

Problem 4. Give an example of a topological space X such that there are disjoint closed sets A and B in X such that there is no continuous $f : X \to [0, 1]$ such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$.

Problem 5. State and prove the **Baire Category Theorem**.

Problem 6. Let $C \subset [0, 1]$ be the standard Cantor set. Let $\{x_i\}_{i=1}^{\infty}$ be any countable subset of $\mathbb{R}$. Show that $\mathbb{R} \setminus \bigcup_{i=1}^{\infty} (x_i + C)$ is dense in $\mathbb{R}$.

Problem 7. Let C be the standard Cantor set. Show that there is a continuous function $f : C \to [0, 1]$ which is onto.

Problem 8. Let X be a metric space with metric d. Show that X is a normal space.

Problem 9. Let X be the space obtained as the quotient space of $[0, 1]$ with the decomposition $\mathscr{D} = (0, 1) \cup \{\{0, 1\}\}$. That is, X is the space that you get identifying 0 and 1 in $[0, 1]$. Show that $[0, 1]/\mathscr{D} \approx S^1$.

Problem 10. Show that there is no function $f : \mathbb{N} \to [0, 1]$ which is onto where $\mathbb{N}$ is the set of positive integers.