Problem 1. Show that $K \subset \mathbb{R}$ is compact if and only if $K$ is closed and bounded.

Problem 2. State and prove the Contraction Mapping Theorem.

Problem 3. Let $X$ be a connected space and suppose that $f : X \to Y$ is continuous and onto. Show that $Y$ is connected.

Problem 4. Give an example of a topological space $X$ such that there are disjoint closed sets $A$ and $B$ in $X$ such that there is no continuous $f : X \to [0, 1]$ such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$.

Problem 5. State and prove the Baire Category Theorem.

Problem 6. Let $C \subset [0, 1]$ be the standard Cantor set. Let $\{x_i\}_{i=1}^\infty$ be any countable subset of $\mathbb{R}$. Show that $\mathbb{R} \setminus \bigcup_{i=1}^\infty (x_i + C)$ is dense in $\mathbb{R}$.

Problem 7. Let $C$ be the standard Cantor set. Show that there is a continuous function $f : C \to [0, 1]$ which is onto.

Problem 8. Let $X$ be a metric space with metric $d$. Show that $X$ is a normal space.

Problem 9. Let $X$ be the space obtained as the quotient space of $[0, 1]$ with the decomposition $\mathcal{D} = (0, 1) \cup \{0, 1\}$. That is, $X$ is the space that you get identifying 0 and 1 in $[0, 1]$. Show that $[0, 1]/\mathcal{D} \approx S^1$.

Problem 10. Show that there is no function $f : \mathbb{N} \to [0, 1]$ which is onto where $\mathbb{N}$ is the set of positive integers.