MTG 5316/4302 PRACTICE FINAL EXAM

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Work all problems and show all work. Each problem is worth ten points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and calculations that are not correct.

Problem 1. Suppose that $X = \mathbb{R}^n \setminus A$ where $A = \{x_i\}_{i=1}^{\infty}$ is any countable subset of $\mathbb{R}^n$ and $n \geq 2$. Show that there is an embedding of S^{n-1} into X.

Problem 2. Suppose that $X = \mathbb{R}^n \setminus A$ where $A = \{x_i\}_{i=1}^{\infty}$ is any countable subset of $\mathbb{R}^n$ and $n \geq 2$. Show that X is arcwise connected.

Problem 3. Suppose that X is a compact connected metric space. Suppose that X is also locally connected. Show that X is arcwise connected.

Problem 4. Suppose that X is a compact metric space. Let C be the Cantor set. Show that there is a continuous map $f : C \to X$ which is onto.

Problem 5. Let A and B be any two countable dense subsets of $\mathbb{R}^n$. Show that there is a homeomorphism $h : \mathbb{R}^n \to \mathbb{R}^n$ such that $h(A) = B$.

Problem 6. Suppose that X is any compact connected locally connected metric space. Show that there is a continuous map $f : [0, 1] \to X$ such that f is onto.

Problem 7. A component of a space X is a maximal connected subset. Show that if A is a component of X, then A is closed.

Problem 8. Show that if A and B are components of X, then either $A = B$ or $A \cap B = \emptyset$.

Problem 9. Let X be a compact metric space with uncountably many components. Show that there is a continuous map $f : X \to C$ which is onto where C is the Cantor set.

Problem 10. Let A and B be disjoint components of X, a compact Hausdorff space. Show that there is a set $U \subset X$ such that $A \subset U$ and $U \cap B = \emptyset$ such that U is both open and closed.