MTG 5316/4302 FALL 2018 PRACTICE FINAL

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1. Major Theorems and Proofs

In this section state and prove each theorem. The proofs should be complete without being tedious.

Problem 1. Suppose that X is a Hausdorff space. Suppose that A and B are disjoint compact subsets of X. Show that there are disjoint open sets U and V in X with $A \subset U$ and $B \subset V$.

Problem 2. State and prove the **Contraction Mapping Theorem**.

Problem 3. State and prove the **Baire Category Theorem** for compact Hausdorff spaces.

Problem 4. State and prove the **Tietze Extension Theorem** for normal spaces. State and assume **Urysohn's Lemma** for normal spaces.

Problem 5. Suppose that X is a compact metric space and Y a metric space. Suppose that $f : X \to Y$ is continuous. Show that f is uniformly continuous.

2. Examples and Minor Proofs

In this section, give brief examples, counterexamples, or quick proofs.

Problem 6. Show that if $A \subset \mathbb{R}$ is connected. Show that A is an interval.

Problem 7. Is it possible to find a continuous function $f : [0, 1] \to C$ which is onto where C is the Cantor set.

Problem 8. Define the **Cantor set**.

Problem 9. Define a continuous function $f : C \to I^2$ which is onto where C is the Cantor set and I^2 is the unit square.

Problem 10. Show that the Cantor set is uncountable.

Problem 11. Give a brief proof that if X is any set, there is no function $f : X \to \mathscr{P}(X)$ which is onto. Here $\mathscr{P}(X)$ is the **power set** of X or the set of all subsets of X.

Problem 12. Let U be an open subset of $\mathbb{R}^n$. Show that if U is connected, then for every $a \neq b \in U$, there is a continuous $f : [0, 1] \to U$ such that f is one-to-one with $f(0) = a$ and $f(1) = b$.

Problem 13. State the **Chain Connectedness Theorem** for connected sets.

Problem 14. Give an example of a connected space that is not arcwise connected.

Problem 15. What does it mean for a space X to be **first countable**? Show that every metric space is first-countable.