1. Major Theorems and Proofs

In this section state and prove each theorem. The proofs should be complete without being tedious.

Problem 1. Suppose that $X$ is a Hausdorff space. Suppose that $A$ and $B$ are disjoint compact subsets of $X$. Show that there are disjoint open sets $U$ and $V$ in $X$ with $A \subset U$ and $B \subset V$.

Problem 2. State and prove the Contraction Mapping Theorem.


Problem 5. Suppose that $X$ is a compact metric space and $Y$ a metric space. Suppose that $f : X \to Y$ is continuous. Show that $f$ is uniformly continuous.
2. EXAMPLES AND MINOR PROOFS

In this section, give brief examples, counterexamples, or quick proofs.

**Problem 6.** Show that if $A \subset \mathbb{R}$ is connected. Show that $A$ is an interval.

**Problem 7.** Is it possible to find a continuous function $f : [0, 1] \to C$ which is onto where $C$ is the Cantor set.

**Problem 8.** Define the **Cantor set**.

**Problem 9.** Define a continuous function $f : C \to I^2$ which is onto where $C$ is the Cantor set and $I^2$ is the unit square.

**Problem 10.** Show that the Cantor set is uncountable.

**Problem 11.** Give a brief proof that if $X$ is any set, there is no function $f : X \to \mathcal{P}(X)$ which is onto. Here $\mathcal{P}(X)$ is the **power set** of $X$ or the set of all subsets of $X$.

**Problem 12.** Let $U$ be an open subset of $\mathbb{R}^n$. Show that if $U$ is connected, then for every $a \neq b \in U$, there is a continuous $f : [0, 1] \to U$ such that $f$ is one-to-one with $f(0) = a$ and $f(1) = b$.

**Problem 13.** State the **Chain Connectedness Theorem** for connected sets.

**Problem 14.** Give an example of a connected space that is not arcwise connected.

**Problem 15.** What does it mean for a space $X$ to be **first countable**? Show that every metric space is first-countable.