FINAL, MTG 5317/4303 SPRING 2019, APRIL 29, 2019, KEESLING

NAME ____

Work all problems. Each problem is worth 10 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. State the **Tychonoff Theorem** for a collection of compact topological spaces.

Problem 2. Let $n \geq 2$. Consider $\mathbb{S}^n/\{x, -x\} = \mathbb{P}^n$. Show that $\pi_1(\mathbb{P}^n) \cong \mathbb{Z}_2$.

Problem 3. State what it means for a topological space to be **Lindelöf**. Show that if X is regular and Lindelöf then it is normal.

Problem 4. Prove the **Alexander Subbase Theorem**.

Problem 5. Suppose that X is a compact metric AR. Show that for any Z and any pair of continuous functions $f, g : Z \to X$, f and g are homotopic.

Problem 6. Let $f, g : X \to \mathbb{R}^n$ be two continuous maps. Show that f and g are homotopic.

Problem 7. Let $p : \mathbb{R} \to \mathbb{S}^1$ such that $p(t) = \exp(2\pi i\, t)$. Suppose that $f : [0,1] \to \mathbb{S}^1$ is a loop with $f(0) = f(1) = 1 \in \mathbb{S}^1$. Show that there is an $\tilde{f} : [0,1] \to \mathbb{R}$ such that $f \equiv p \circ \tilde{f}$.

Problem 8. State what it means for a toral automorphism $f : \mathbb{T}^n \to \mathbb{T}^n$ to be **hyperbolic**. Show that **Arnold cat map** given by the matrix

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

is hyperbolic from $\mathbb{T}^2$ to $\mathbb{T}^2$.

Problem 9. State and prove the **Brouwer Fixed Point Theorem** for I^2.

Problem 10. Use the **Brouwer Characterization of the Cantor Set** to show that if X is a compact metric space, then there is a continuous mapping $f : C \to X$ that is onto.