QUIZ 1 MTG 5317/4303 SPRING 2019

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Problem 1. Let $f : I^2 \to I^2$ be a continuous function. Show that there is an $x \in I^2$ such that $f(x) = x$. This is called the **Brouwer Fixed Point Theorem** for I^2. The theorem is also true for I^n for all $n \geq 1$.

Problem 2. Suppose that $f : \mathbb{S}^1 \to \mathbb{S}^1$ is continuous. Show that f is homotopic to $z^n : \mathbb{S}^1 \to \mathbb{S}^1$ for some $n \in \mathbb{Z}$. This is the **degree** of f.

Problem 3. Let $f : \mathbb{S}^n \to \mathbb{S}^n$ and suppose that $f(x) = -f(-x)$ for all $x \in \mathbb{S}^n$. Then we say that f is an **antipodal map**. Let $f : \mathbb{S}^1 \to \mathbb{S}^1$ be an antipodal map. Show that f is homotopic to z^k for some odd integer k.

Problem 4. Let $f : \mathbb{S}^2 \to \mathbb{R}^2$ be a continuous function. Show that there is an $x \in \mathbb{S}^2$ such that $f(x) = f(-x)$. This is called the **Borsuk-Ulam Theorem** for $n = 2$. The theorem is also true for all $n \geq 1$.