Problem 1. Let $f : I^2 \to I^2$ be a continuous function. Show that there is an $x \in I^2$ such that $f(x) = x$. This is called the Brouwer Fixed Point Theorem for $I^2$. The theorem is also true for $I^n$ for all $n \geq 1$.

Problem 2. Suppose that $f : S^1 \to S^1$ is continuous. Show that $f$ is homotopic to $z^n : S^1 \to S^1$ for some $n \in \mathbb{Z}$. This is the degree of $f$.

Problem 3. Let $f : S^n \to S^n$ and suppose that $f(x) = -f(-x)$ for all $x \in S^n$. Then we say that $f$ is an antipodal map. Let $f : S^1 \to S^1$ be an antipodal map. Show that $f$ is homotopic to $z^k$ for some odd integer $k$.

Problem 4. Let $f : S^2 \to \mathbb{R}^2$ be a continuous function. Show that there is an $x \in S^2$ such that $f(x) = f(-x)$. This is called the Borsuk-Ulam Theorem for $n = 2$. The theorem is also true for all $n \geq 1$. 