QUIZ 2 MTG 5317/4303 SPRING 2019

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Problem 1. Let $f : \mathbb{T}^2 \to \mathbb{T}^2$ be the **Arnold cat map**. By definition f is an automorphism of $\mathbb{T}^2$ such that $f_* : \pi_1(\mathbb{T}^2) \cong \mathbb{Z}^2 \to \pi_1(\mathbb{T}^2) \cong \mathbb{Z}^2$ is the matrix

$$f_* = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

What are the eigenvalues and eigenvectors of this matrix? How do these affect the map $f : \mathbb{T}^2 \to \mathbb{T}^2$?

Problem 2. Let $f : \mathbb{T}^2 \to \mathbb{T}^2$ be the Arnold cat map, Consider $\mathbb{T}^2$ as the quotient of $[0,1] \times [0,1]$ under the exponential map. Show that the periodic points of f are the image of the rational points under the quotient map.

Problem 3. Let $f : \mathbb{T}^n \to \mathbb{T}^n$ be such that f_* is given by the matrix

$$\begin{bmatrix} m_{11} & \cdots & m_{1n} \\ m_{21} & \cdots & m_{2,n} \\ \vdots & \ddots & \vdots \\ m_{n1} & \cdots & m_{nn} \end{bmatrix}.$$

What condition is necessary for f to be a homeomorphism? When the condition is met is there a homeomorphism g homotopic to f? What is the significance of $\det(M)$?

Problem 4. Let $f : \mathbb{T}^2 \to \mathbb{T}^2$ be the Arnold cat map. How would you find a point of period three? Of period one hundred?

Problem 5. Let $f : \mathbb{T}^2 \to \mathbb{T}^2$ be an automorophism such that f_* is given by the matrix

$$f_* = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}.$$

Analyze the behavior this automorphism.

Problem 6. Let $f : \mathbb{T}^n \to \mathbb{T}^n$ be an automorophism such that f_* is given by the matrix

$$M = \begin{bmatrix} m_{11} & \cdots & m_{1n} \\ m_{21} & \cdots & m_{2,n} \\ \vdots & \ddots & \vdots \\ m_{n1} & \cdots & m_{nn} \end{bmatrix}.$$

We say that f is **hyperbolic** provided that for every eigenvalue λ of M $|\lambda| \neq 1$. What are the periodic points of f?