QUIZ 3 MTG 5317/4303 SPRING 2019

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Problem 1. State and prove the **Brouwer Fixed Point Theorem** for dimension 2.

Problem 2. State and prove the **Borsuk-Ulam Theorem** for dimension 2.

Problem 3. Show that $\pi_1(\mathbb{S}^2) = \{1\}$ using the **Seifert-van Kampen Theorem**.

Problem 4. Show that $\pi_1(\mathbb{T}^2) \cong \mathbb{Z} \times \mathbb{Z}$ using the **Seifert-van Kampen Theorm**.

Problem 5. Let $n \geq 2$ be an integer. Determine a space X such that $\pi_1(X) \cong \mathbb{Z}_n$ using the **Seifert-van Kempen Theorem**.

Problem 6. Suppose that the group G has a representation as $G = \langle g_1, g_2, \ldots, g_n \,:\, r_1, r_2, \ldots, r_m \rangle$. Determine a space X whose fundamental group is G. Use the **Siefert-van Kampen Theorem**.