

## QUIZ 4 MTG 5317/4303 SPRING 2019

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**Problem 1.** Suppose that  $n > 1$ . Show that  $\mathbb{R}^n$  is not homeomorphic to  $\mathbb{R}$ .

**Problem 2.** Suppose that  $n > 2$ . Show that  $\mathbb{R}^n$  is not homeomorphic to  $\mathbb{R}^2$ .

**Problem 3.** Suppose that  $X$  is a regular Lindelöf space. Show that  $X$  is normal.

**Problem 4.** Let  $X$  be a locally compact Hausdorff space. Describe the topology of the one-point compactification,  $X \cup \{\infty\}$ , and show that this space is compact and Hausdorff.

**Problem 5.** Suppose that  $X$  is completely regular. Show that there is a compact Hausdorff space  $\beta X$  that contains  $X$  as a dense subset such that for every  $f : X \rightarrow [0, 1]$  that is continuous, there is a continuous  $\beta f : \beta X \rightarrow [0, 1]$  such that  $\beta f|_X \equiv f : X \rightarrow [0, 1]$ .