

QUIZ 5 MTG 5317/4303 SPRING 2019

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Problem 1. State and prove the **Alexander Subbase Theorem**.

Problem 2. State and prove the **Tychonoff Theorem**.

Problem 3. Show that the Tychonoff Theorem implies the Axiom of Choice.

Problem 4. Suppose that X is completely regular. Show that if βX is metrizable, then X compact metric and $\beta X = X$.

Problem 5. Let $f = z^2 : \partial I^2 \rightarrow \mathbb{S}^1$. Show that I^2/f is homeomorphic to $\mathbb{P}^2 = \mathbb{S}^2/\{x, -x\}$.

Problem 6. Let $S = \mathbb{T}^2 \# \mathbb{T}^2$ be the connected sum of two tori. What is $\pi_1(S)$?