# TEST 1, MTG 5317/4303 SPRING 2019, MARCH 15, 2019

JAMES KEESLING

**NAME** ________________________________________

Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

**Problem 1.** Let $f, g : X \to \mathbb{R}^n$ be two continuous maps. Show that $f$ and $g$ are homotopic.

**Problem 2.** Let $p : \mathbb{R} \to \mathbb{S}^1$ such that $p(t) = \exp(2\pi i\, t)$. Suppose that $f : [0,1] \to \mathbb{S}^1$ is a loop with $f(0) = f(1) = 1 \in \mathbb{S}^1$. Show that there is an $\tilde{f} : [0,1] \to \mathbb{R}$ such that $f \equiv p \circ \tilde{f}$.

**Problem 3.** State what it means for a toral automorphism $f : \mathbb{T}^n \to \mathbb{T}^n$ to be **hyperbolic**. Show that **Arnold cat map** given by the matrix

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

is hyperbolic from $\mathbb{T}^2$ to $\mathbb{T}^2$.

**Problem 4.** State and prove the **Brouwer Fixed Point Theorem** for $I^2$.

**Problem 5.** State the **Brouwer Characterization of the Cantor Set**.