**TEST 2, MTG 5317/4303 SPRING 2019, APRIL 24, 2019, KEESLING**

**NAME** ________________________________________

Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

**Problem 1.** State and prove the **Tychonoff Theorem** for a collection of compact topological spaces.

**Problem 2.** Let $n \geq 2$. Consider $\mathbb{S}^n/\{x, -x\} = \mathbb{P}^n$. Show that $\pi_1(\mathbb{P}^n) \cong \mathbb{Z}_2$.

**Problem 3.** State what it means for a topological space to be **Lindelöf**. Show that if $X$ is a metric space with a countable base $\mathscr{B}$, then $X$ is Lindelöf.

**Problem 4.** State the **Alexander Subbase Theorem**.

**Problem 5.** Suppose that $X$ is a compact metric AR. Show that for any $Z$ and any pair of continuous functions $f, g : Z \to X$, $f$ and $g$ are homotopic.